IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

Stochastic model in manpower planning using shock model approach

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ABSTRACT

In any organization the exit of personnel occurs, consequent to the announcement of policy decisions regarding wages, incentives and targets. Such a phenomenon is quite common in market organization. The exit of personnel which otherwise known as wastage leads to the depletion of manpower available with the organization. In this paper expected time to cross the threshold level of cumulative loss of manpower and its varaiance are obtained under the assumption that an organization has two grades of personnel and that the loss of manpower in one category is compensated by utilization of manpower available in the other category in order to sustain the activities intact.

INTRODUCTION

In marketing organizations it is a usual phenomenon that exit of personnel happens every time the policy decisions regarding revision of wages, incentives and revised sales targets are announced. This in turn leads to depletion of manpower which can be conceptualized interms of man hours. It would be uneconomical to go in for frequent recruitments in view of the cost involved in the same. A detailed discussion of the cost pertaining to recruitment is seen in Poornachandra Rao (1990). Hence the organization goes for recruitments as and when the cumulative loss of manpower on successive occasions crosses a random threshold level beyond which the marketing activities would adversely be affected. Using the shock model approach and cumulative damage processes the expected time to breakdown of the organization, which in other words is the expected time to recruitment and its variance have been obtained by Parthasarathy (2002). One can also refer Esary et al., (1973), Bartholomew,D.J. and Forbes,A.F (1979).

Introducing two grades of marketing personnel in the organization, the expected time to recruitment and its variance are obtained allowing for the mobility of manpower from one category to the other where there is more of depletion. The two grades are assumed to have their own threshold levels. Obviously the breakdown occurs only when the total depletion crosses the maximum of the two threshold levels. Sathyamoorthyand Parthasarathy (2002) discussed that the threshold levels for the two grades are y_1 and y_2 respectively and the breakdown point occurs when the total loss of manpower due to wastage crosses $Y = \text{Max}(Y_1, Y_2)$. This is by the virtue of the fact that the transfer of personnel from one grade to the other grade to the other grade is possible so that the time to breakdown of the system can be elongated. However, such a kind of an assumption may not hold always and so transfers of personnel from are grade to the other is not permitted. For example in a two grade organization, one grade comprising the administrative staff/personnel and another grade comprising of technical personnel of the production venue, if depletion of staff occurs in both grades due to policy revisions, the breakdown of the organization as industry is imminent. This occurs if the depletion crosses a threshold level. The breakdown occurs as and when the depletion crosses the threshold level in any one of the two grades first. Under such an assumption this paper attempt to find the the expected time to the breakdown of the organization in other words the expected time to recruitment and its variances.

ASSUMPTIONS OF THE MODEL

1. The organization comprises of two grades of personnel.
2. The transfer of manpower from one grade to other is not permitted.
3. Each grade as its random threshold and if the loss of manpower crosses that level the breakdown the organization occurs and recruitment becomes necessary.
4. The interval arrival times between policy decisions are taken to be i.i.d. r.v's.

NOTATIONS

X_i : a continuous random variable denoting the amount of loss of manpower (in man hours) caused to the system due to the ith occasion of policy announcement and X_i's are i.i.d. denoted as X.

Y_1, Y_2: continuous random variables denoting the threshold levels for the two grades, and $Y_1 \sim \exp(\theta_1)$ and $Y_2 \sim \exp(\theta_2)$.

g(.) : the probability density function of X.

g*(.): Laplace transform of g(.).

g_k (.) : the k-fold convolution of g(.) i.e. p.d.f. of $\sum_{i=1}^{k} X_i$.

T_1 : Time to breakdown of the system due to depletion in the first grade.

T_2 : Time to breakdown of the system due to depletion in the second grade.

T : a continuous r.v. denoting the time to breakdown of the system or recruitment and T = min (T_1, T_2).

f(.) : p.d.f. of r.v. denoting interarrival times between the successive policy decisions with corresponding c.d.f. F(.)

F_k(.): the k-fold convolution of F(.).

V_k(t): tradability that there are exactly k decisions

RESULT

It may be observed that this particular situation is one which is very much similar to the concept of competing exponential distribution. The competing exponential distribution arises when E_1 and E_2 are two independent events which with compete each other for the first occurrence. The two independents r.v's T_1 and T_2 represent the occurrence times E_1 and E_2 respectively.

T = Min (T_1, T_2). In this case the distribution function of Y can be derived as follows:

$$S(y) = P[Y > y] = P[\text{Min}(Y_2, Y_1)] \quad (1)$$

Therefore, $S(y) = e^{-(\mu_1 + \mu_2)y}$, so we can write, $S(x) = e^{-(\mu_1 + \mu_2)x}$

The survivor function S (t) is

P (T > t) = Probability that the system survives beyond 't'

$$= \sum_{k=0}^{\infty} P \text{ (there are exactly k instants of exits in (0, t]}$$

* P (the system does not fail in (0, t])

$$= \sum_{k=0}^{\infty} V_k(t)\; P\,[\sum_{i=1}^{k} X_i < \min(Y_1, Y_2)\,]$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \int_o^{\infty} g_k(x) \left[e^{-(\mu_1+\mu_2)x}\right] dx$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\; [g^*(\mu_1 + \mu_2)]^k \quad (2)$$

P (T < t) = L (t)

$$= [1 - g^*(\mu_1+\mu_2)] \sum_{k=1}^{\infty} [g^*(\mu_1 + \mu_2)]^{k-1} F_k(t)$$ on simplification

Taking Laplace transform of L(t), we get

$$L^*(s) = [1 - g^*(\mu_1+\mu_2)] \sum_{k=1}^{\infty} [g^*(\mu_1 + \mu_2)]^{k-1} f_k^*(s)$$

$$= [1 - g^*(\mu_1+\mu_2)]\; f^*(s) \sum_{k=1}^{\infty} [g^*(\mu_1 + \mu_2)\; f^*(s)]^{k-1}$$

where $[f^*(s)]^k$ is Laplace transform of F_k (t) since Xi's are i.i.d.

$$L^*(s) = \frac{[1 - g^*(\mu_1 + \mu_2)] f^*(s)}{1 - g^*(\mu_1 + \mu_2) f^*(s)}$$ on simplification. (3)

Special case

Let f(.) ~ exp (c) then $f^*(s) = \frac{c}{c+s}$

$$g^*(\mu_1+\mu_2) = \frac{\lambda}{\lambda + \mu_1 + \mu_2}$$

we get,

$$L^*(s) = \frac{\left[1 - \frac{\lambda}{\lambda + \mu_1 + \mu_2}\right] \frac{c}{c+s}}{1 - \frac{\lambda}{\lambda + \mu_1 + \mu_2} \frac{c}{c+s}}$$

Using (5 and 6) we get

$$E(T) = \frac{1}{c}\left[\frac{[\mu_1 + \mu_2 + \lambda]}{\mu_1 + \mu_2}\right] \quad (4)$$

$$E(T^2) = \frac{2}{c^2}\left[\frac{[\mu_1 + \mu_2 + \lambda]}{\mu_1 + \mu_2}\right]^2 \qquad (5)$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$= \frac{1}{c^2}\left[\frac{[\mu_1 + \mu_2 + \lambda]}{\mu_1 + \mu_2}\right]^2 \qquad (6)$$

NUMERICAL ILLUSTRATION

In this model based on competing exponential distribution, the Table 1 and the corresponding Figure-1 indicate that as C increases expectation of T decreases at fast rate. It is so, if λ is also increases, however it may be observed that E(T) undergoes a rapid change as C increases and it is very much significant when compared with the previous model.

The behavior of variance for the changes in C for fixed λ as well as for different λ values could be observed in Table 2 and Figure-2. It is striking to note that here also the variance is initially large when C is small but undergoes at very fast change as C increases. However, the variance convergence the same set of values as C increases.

Table 1

C	Expectation of T		
	$\theta_1 = 0.2$, $\theta_2 = 0.1$, $\lambda = 1.0$	$\theta_1 = 0.2$, $\theta_2 = 0.1$, $\lambda = 5.0$	$\theta_1 = 0.2$, $\theta_2 = 0.1$, $\lambda = 9.0$
1	4.33333	17.6666	31.0000
2	1.0833	4.4167	7.7500
3	0.4815	1.9629	3.4445
4	0.2708	1.1042	1.9375
5	0.1733	0.7067	1.2400
6	0.1204	0.4907	0.8611
7	0.0884	0.3605	0.6327
8	0.0677	0.2760	0.4844
9	0.0535	0.2181	0.3827
10	0.0433	0.1767	0.3100

Table 2

C	Variance of T		
	$\theta_1 = 0.2$, $\theta_2 = 0.1$, $\lambda = 1.0$	$\theta_1 = 0.2$, $\theta_2 = 0.1$, $\lambda = 5.0$	$\theta_1 = 0.2$, $\theta_2 = 0.1$, $\lambda = 9.0$
1	18.7778	312.1110	961.0000
2	1.1736	19.5069	60.0625
3	0.2318	3.8532	11.8642
4	0.0734	1.2192	3.7539
5	0.0304	0.4994	1.5376
6	0.0145	0.2408	0.7415
7	0.0078	0.1299	0.4003
8	0.0046	0.0762	0.2346
9	0.0029	0.0476	0.1465
10	0.0019	0.0312	0.0961

Figure - 1

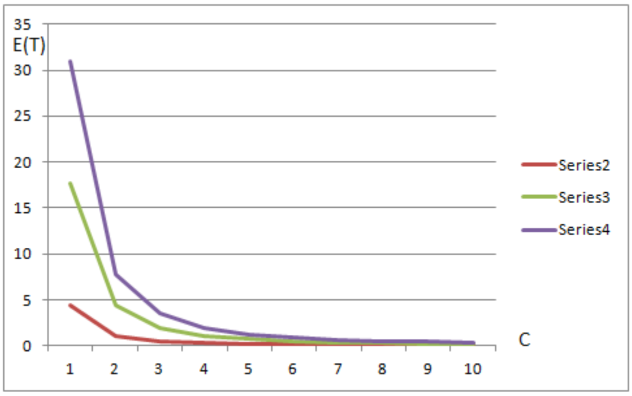
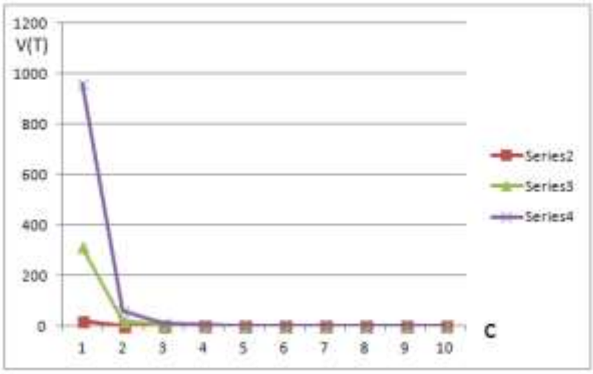


Figure - 2



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